Gimbal Lock

- Experiencing gimbal lock with a 3D tripod head
  - Adjust starting from object (camera)
    - roll, pitch, yaw around axes of parent’s (tripod) coord sys
  - Adjust starting from parent (tripod)
    - yaw, pitch, roll around axes of object’s (camera) coord sys
Demo: The Perils of Roll, Pitch, & Yaw

Gimbal Lock

- A real physical problem in aircraft/spacecraft navigation systems with physical gimbals
- A real conceptual problem in 3D user interfaces
  - There is always an ordered set of rotations about the given axes that will produce a desired rotation
    - But, it might not involve an intuitive incremental change from the current set of rotations
**Axis of Rotation and Angle**

- Can represent any rotation as an axis of rotation and an angle
  - Avoids gimbal lock
- But, matrices, rotations about an ordered set of axes, axis of rotation and angle all have problems interpolating between two rotations
  - Smoothly
  - Along shortest path

**Quaternions**  
*W. Hamilton, 1843*

- A quaternion is
  - A four-tuple $xi + yj + zk + w$, where $i, j, k$ are imaginary
    - $i^2 = j^2 = k^2 = ijk = -1$
    - $ij = k, ji = -k$
    - $jk = i, kj = -i$
    - $ki = j, ik = -j$
  - $q = (v, w) = v_x i + v_y j + v_z k + w$
    - $v$ is a vector, $w$ is a scalar
Quaternions W. Hamilton, 1843

- A unit quaternion ($|q| = 1$) represents rotation about normalized rotation axis $u$ by angle $\theta$
  - $v = (\sin (\theta / 2)) \, u$
  - $w = \cos(\theta / 2)$

$$|q| = \sqrt{v_x^2 + v_y^2 + v_z^2 + w^2}$$

A unit quaternion is a point on a 4D unit sphere

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Quaternions W. Hamilton, 1843

- Converting to a rotation matrix

$$\begin{bmatrix}
1 - 2y^2 - 2z^2 & 2xy + 2wz & 2xz - 2wy & 0 \\
2xy - 2wz & 1 - 2x^2 - 2z^2 & 2yz + 2wx & 0 \\
2xz + 2wy & 2yz - 2wx & 1 - 2x^2 - 2y^2 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}$$

- Multiplying quaternions

$$q_1 \cdot q_2 = (v_1, w_1) \cdot (v_2, w_2) = (w_1 v_2 + w_2 v_1 + v_1 \otimes v_2, w_1 w_2 - v_1 v_2)$$

- Product of two unit quaternions corresponds to composition of their rotations (associative, but not commutative)
Quaternions  W. Hamilton, 1843

- Spherical Linear Interpolation (SLERP)
  - Smoothly interpolates two quaternions as points on 4D unit sphere
    - Constant angular velocity along great circle between two points on 4D unit sphere

- Support in Unity
  - You do not have to implement the math yourself!
  - https://unity3d.com/learn/tutorials/modules/intermediate/scripting/quaternions

Inside the Demo…